Mobile Tracking and Parameter Learning in Unknown Non-line-of-sight Conditions

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Abstract – *This paper studies the mobile tracking problem in mixed line-of-sight (LOS) and non-line-of-sight (NLOS) conditions, where the statistics of NLOS error is Gaussian with fixed but unknown mean and variance. A Rao-Blackwellized particle filtering and parameter learning method (RBPF-PL) is proposed, in which the particle filtering with optimal trial distribution is first applied to estimate the posterior density of sight conditions, then the decentralized extended Kalman filter (EKF) is used to estimate the mobile state. In the parameter learning step, using the conjugate prior distribution on the unknown parameters, the posterior distribution of unknown parameters is further updated according to the sufficient statistics. Simulation results show the RBPF-PL method is effective to infer the unknown NLOS parameter and could achieve good tracking performance using small number of particles.*

Keywords: mobile tracking, non-line-of-sight, particle filtering, Rao-Blackwellized, parameter learning

1 Introduction

Precise positioning in non-line-of-sight (NLOS) conditions is a tough task for many wireless positioning systems. In typical NLOS circumstances, like urban canyons, the direct path between transceiver has been blocked by buildings and other obstacles. The propagation waves' path may be lengthened due to reflection, refraction and scattering. Based on the measurement of the scattered signals, huge localization errors will be introduced.

Methods proposed to deal with the NLOS problem in mobile tracking applications generally exploit the measurements in time series to mitigate the NLOS errors. To name a few, these methods include two-step Kalman filtering techniques for smoothing range measurements and mitigating NLOS errors [1], a Kalman based interacting multiple model (IMM) smoother [2], grid based Bayesian estimation [3], particle filtering (PF) [4], a modified extended Kalman filter (EKF) bank [5], the improved Rao-Blackwellized particle filtering (RBPF) [6], joint particle filter and unscented Kalman filtering (UKF) method [7], etc. A posterior Cramér-Rao lower bound is further investigated in [8].

Prior research assumes a complete knowledge of statistics of NLOS errors, including the statistical parameters. The exact knowledge of the error statistics, and especially the parameters is not a plausible assumption in many practical situations .

In this study, we consider the mobile tracking problem in the mixed LOS/NLOS conditions under the assumption that the statistics of LOS error (usually treated as the measurement noise) is known and conforms to zero mean Gaussian distribution, while the statistics of NLOS error is Gaussian with its mean and variance fixed static but unknown.

To tackle the problem of sequential state estimation with the inference of unknown but static parameters, Liu and West assumed an artificial dynamic evolution for the unknown parameter vector, which could be further included in the state vector [9]. However, such treatment would enlarge the estimation (co)variance of the unknown parameter. Djurić et al [10] suggested the use of density-assisted particle filters (DAPFs) as a viable alternative to jointly estimate the sequential state and the parameter without introducing an artificial dynamic model for the static parameters. However, in our situation, because of the Markov property of the sight condition, the derivation of the density update function for all the state variables and the static parameter is not an easy task. Storvik [11] proposed to marginalize the static parameters out of the posterior distribution such that only the state vector needs to be considered. We shall adopt this way of treating the static parameters in the sequential estimation and propose a Rao-Blackwellized particle filtering with parameter learning (RBPF-PL) method, which applies particle filter to estimate the posterior density of sight conditions, then uses the analytical method to estimate the mobile state. Based on the above two-step estimation, the distribution of the unknown parameters is updated by sufficient statistics.

The paper is organized as follows: Section 2 presents the system model and formulates the problem of mobile tracking in the mixed LOS/NLOS conditions. Section 3 considers the problem within the Bayesian framework. In Section 4, the RBPF-PL method is further described in detail. Numerical results and performance comparison are presented and discussed in Section 5. Section 6 draws some conclusions.

2 System model

Assume a mobile of interest moves on a two-dimensional Cartesian plane. The state at time instant t_k is defined as the vector $\mathbf{x}_k = [x_k, y_k, \dot{x}_k, \dot{y}_k]^T$, where $[x_k, y_k]^T$ corresponds to the east and north coordinates of the mobile position; $[\dot{x}_k, \dot{y}_k]^T$ are the corresponding velocities. The mobile state with random acceleration can be modeled as [12]:

$$\mathbf{x}_{k+1} = \Phi_k \mathbf{x}_k + \mathbf{w}_k, \tag{1}$$

where the transition matrix Φ_k models the state kinematics. The random process $\mathbf{w}_k$ is a white zero mean Gaussian noise, with covariance matrix Q.

We consider the tracking problem in the context of mobile cellular systems and assume the range is measured by time-of-arrival (TOA) method. Under possible NLOS propagation condition between the mobile station (MS) and the base station (BS), the distance measured at time t_k is

$$z_{i,k} = d_{i,k} + v(s_{i,k}), \tag{2}$$

where $d_{i,k} \triangleq h_{i,k}(\mathbf{x}_k) = \sqrt{(x_k - x_{\mathrm{bs}_i})^2 + (y_k - y_{\mathrm{bs}_i})^2}$, represents the true distance between the mobile position $[x_k, y_k]^T$ and the location of the ith BS $[x_{\mathrm{bs}_i}, y_{\mathrm{bs}_i}]^T$, $i \in \{1, 2, \ldots, M\}$ and M is the number of BSs.

Boolean variable $s_{i,k} \in \{0, 1\}$ represents LOS/NLOS condition between the MS and BS_i, with $s_{i,k} = 0$ for LOS and $s_{i,k} = 1$ for NLOS. In mobile tracking, the sight conditions undergo dynamical transitions, which can be further modeled as a time-homogeneous first-order Markov chain $s_{i,k} \sim \mathrm{MC}(\pi_i, \mathrm{A}_i)$ with initial probability vector π_i and the transition probability matrix

$$\mathrm{A}_i = \begin{bmatrix} p_0 & 1 - p_0 \\ 1 - p_1 & p_1 \end{bmatrix},$$

where $p_0 = \mathrm{P}(s_{i,k} = 0 | s_{i,k-1} = 0)$ and $p_1 = \mathrm{P}(s_{i,k} = 1 | s_{i,k-1} = 1)$.

Assume that the measurement noise in the LOS condition conforms to zero mean Gaussian distribution $\mathrm{N}(0, \sigma_n^2)$, while the NLOS error is modeled as a biased Gaussian distribution $\mathrm{N}(\mu_{\mathrm{NLOS}}, \sigma_{\mathrm{NLOS}}^2)$ [1, 2, 5]. Thus, $v(s_{i,k}) \sim \mathrm{N}(m(s_{i,k}), \mathrm{R}(s_{i,k}))$ and

$$\begin{aligned} m(s_{i,k}) &= s_{i,k} \mu_{\mathrm{NLOS}} \\ \mathrm{R}(s_{i,k}) &= \sigma_n^2 + s_{i,k} \sigma_{\mathrm{NLOS}}^2. \end{aligned} \tag{3}$$

In this work, we assume that σ_n is known, while $\{\mu_{\mathrm{NLOS}}, \sigma_{\mathrm{NLOS}}^2\}$ are fixed static but unknown.

3 Mobile tracking and parameter learning within Bayesian framework

Denote the total observation sequence up to time t_k as $\mathbf{z}_{1:k}$, where $\mathbf{z}_k \triangleq [z_{1,k}, z_{2,k}, \ldots, z_{M,k}]^T$. For brevity, let $\eta \triangleq \sigma_n^2 + \sigma_{\mathrm{NLOS}}^2$ and $\theta = \{\mu_{\mathrm{NLOS}}, \eta\}$. The problem of mobile tracking in the unknown NLOS conditions is to simultaneously infer the mobile state $\mathbf{x}_k$, the sight condition $\mathbf{s}_k$ and NLOS noise θ from the observation sequence $\mathbf{z}_{1:k}$, which corresponds to computing the joint posterior $p(\mathbf{x}_k, \mathbf{s}_k, \theta | \mathbf{z}_{1:k})$. The analytical solution to the posterior requires high-dimensional integrals. Here, we resort to sequential Monte Carlo method.

Denote $\mathbf{y}_k = \{\mathbf{x}_k, \mathbf{s}_k\}$. Consider the sequential estimation $p(\mathbf{y}_{1:k}, \theta | \mathbf{z}_{1:k})$ within the Bayesian framework:

$$\begin{aligned} p(\mathbf{y}_{1:k}, \theta | \mathbf{z}_{1:k}) &\propto p(\mathbf{z}_k | \mathbf{y}_k, \theta) p(\mathbf{y}_{1:k}, \theta | \mathbf{z}_{1:k-1}) \\ &= p(\mathbf{z}_k | \mathbf{y}_k, \theta) p(\mathbf{y}_k | \mathbf{y}_{k-1}) \\ &\quad \cdot p(\theta | \mathbf{y}_{1:k-1}, \mathbf{z}_{1:k-1}) p(\mathbf{y}_{1:k-1} | \mathbf{z}_{1:k-1}) \end{aligned}$$

Suppose at time t_{k-1} we have samples constituting an approximation to $p(\mathbf{y}_{1:k-1} | \mathbf{z}_{1:k-1})$, i.e,

$$p(\mathbf{y}_{1:k-1} | \mathbf{z}_{1:k-1}) \approx \sum_{j=1}^{N} w_{k-1}^j \delta(\mathbf{y}_{1:k-1} - \mathbf{y}_{1:k-1}^j) \tag{4}$$

With the reception of measurement $\mathbf{z}_k$, we wish to approximate $p(\mathbf{y}_{1:k}, \theta | \mathbf{z}_{1:k})$ with a new set of samples. If the importance density is chosen to factorize such that

$$q(\mathbf{y}_{1:k}, \theta | \mathbf{z}_{1:k}) = q(\mathbf{y}_k, \theta | \mathbf{y}_{1:k-1}, \mathbf{z}_{1:k}) q(\mathbf{y}_{1:k-1} | \mathbf{z}_{1:k-1})$$

and based on the samples in (4), the new particles at time t_k are sampled according to

$$\{\mathbf{y}_k^j, \theta^j\} \sim q(\mathbf{y}_k, \theta | \mathbf{y}_{1:k-1}^j, \mathbf{z}_{1:k})$$

then the weight can be updated as

$$\begin{aligned} w_k^j &\propto \frac{p(\mathbf{y}_{1:k}^j, \theta^j | \mathbf{z}_{1:k})}{q(\mathbf{y}_{1:k}^j, \theta^j | \mathbf{z}_{1:k})} \\ &= \frac{p(\mathbf{z}_k | \mathbf{y}_k^j, \theta^j) p(\mathbf{y}_k^j | \mathbf{y}_{k-1}^j) p(\theta^j | \mathbf{y}_{1:k-1}^j, \mathbf{z}_{1:k-1})}{q(\mathbf{y}_k^j, \theta^j | \mathbf{y}_{1:k-1}^j, \mathbf{z}_{1:k})} w_{k-1}^j \end{aligned} \tag{5}$$

In standard particle filtering, transition priors are utilized as the proposal distribution:

$$\begin{aligned} &q(\mathbf{y}_k, \theta | \mathbf{y}_{1:k-1}^j, \mathbf{z}_{1:k}) \\ &\quad = p(\mathbf{x}_k | \mathbf{x}_{k-1}^j) p(\mathbf{s}_k | \mathbf{s}_{k-1}^j) p(\theta^j | \mathbf{y}_{1:k-1}^j, \mathbf{z}_{1:k-1}). \end{aligned}$$

Thus, the weight update equation (5) can be simplified as:

$$w_k^j \propto w_{k-1}^j p(\mathbf{z}_k | \mathbf{y}_k^j, \theta^j).$$

4 Rao-Blackwellized particle filtering with parameter learning

In standard particle filtering, since $\{\mathbf{x}_k, \mathbf{s}_k, \theta\}$ constitutes a high dimensional state estimation space, a large number of particles should be used to achieve good estimation results, which is computationally expensive. Additionally, using the transition prior as the proposal, which fails to consider the information of current measurements, would easily suffer from "particle impoverishment" problem.

In this section, we present a RBPF-PL method, which only uses particle filtering method to estimate the posterior of sight condition $\mathbf{s}_k$ while applying an analytical method to estimate the mobile state $\mathbf{x}_k$ and updating the parameters of NLOS distribution θ by sufficient statistic information. Since the estimation of $\{\mathbf{x}_k, \theta\}$ largely depends on the accuracy of $\mathbf{s}_k$, in the particle filtering we use the optimal trial distribution to sample the particles. The method is described as follows.

Factorize the posterior $p(\mathbf{x}_k, \mathbf{s}_k, \theta|\mathbf{z}_{1:k})$ according to Bayes' rule:

$$p(\mathbf{x}_k, \mathbf{s}_k, \theta|\mathbf{z}_{1:k}) = p(\mathbf{x}_k|\mathbf{s}_k, \theta, \mathbf{z}_{1:k})p(\mathbf{s}_k, \theta|\mathbf{z}_{1:k}). \tag{6}$$

If $p(\mathbf{s}_k, \theta|\mathbf{z}_{1:k})$ is represented by a set of weighted samples $\{\mathbf{s}_k^j, \theta^j, w_k^j\}_{j=1}^N$, i.e.,

$$p(\mathbf{s}_k, \theta|\mathbf{z}_{1:k}) \approx \sum_{j=1}^{N} w_k^j \delta(\mathbf{s}_k - \mathbf{s}_k^j)\delta(\theta - \theta^j), \tag{7}$$

then the marginal density $p(\mathbf{x}_k|\mathbf{z}_{1:k})$ can be approximately expressed by a mixture of densities:

$$\begin{aligned} p(\mathbf{x}_k|\mathbf{z}_{1:k}) &\approx \sum_{j=1}^{N} w_k^j p(\mathbf{x}_k|\mathbf{s}_k, \theta, \mathbf{z}_{1:k})\delta(\mathbf{s}_k - \mathbf{s}_k^j)\delta(\theta - \theta^j) \\ &= \sum_{j=1}^{N} w_k^j p(\mathbf{x}_k|\mathbf{s}_k^j, \theta^j, \mathbf{z}_{1:k}), \end{aligned} \tag{8}$$

where the mixture component $p(\mathbf{x}_k|\mathbf{s}_k^j, \theta^j, \mathbf{z}_{1:k})$ approximately conforms to Gaussian distribution $\mathrm{N}(\hat{\mathbf{x}}_k^j, \hat{\mathrm{P}}_k^j)$, where

$$\hat{\mathbf{x}}_k^j = \hat{\mathbf{x}}_{k|k-1}^j + \sum_{i=1}^{M} \mathrm{K}_{i,k}^j(z_{i,k} - \hat{z}_{i,k|k-1}^j) \tag{9}$$

$$\hat{\mathrm{P}}_k^j = \left[(\hat{\mathrm{P}}_{k|k-1}^j)^{-1} + \sum_{i=1}^{M}(\mathrm{H}_{i,k}^j)^T \mathrm{R}(s_{i,k}^j)^{-1}\mathrm{H}_{i,k}^j\right]^{-1} \tag{10}$$

$\hat{\mathbf{x}}_{k|k-1}^j$ is the predicted mean of $\mathbf{x}_{k-1}^j$:

$$\hat{\mathbf{x}}_{k|k-1}^j = \Phi\hat{\mathbf{x}}_{k-1}^j \tag{11}$$

and $\hat{\mathrm{P}}_{k|k-1}^j$ is the corresponding predicted covariance:

$$\hat{\mathrm{P}}_{k|k-1}^j = \Phi_{k-1}\hat{\mathrm{P}}_{k-1}^j\Phi_{k-1}^T + \mathrm{Q} \tag{12}$$

The predicted mean of measurement $\hat{z}_{i,k|k-1}^j$ is

$$\hat{z}_{i,k|k-1}^j = h_i(\hat{\mathbf{x}}_{k|k-1}^j) + m(s_{i,k}^j) \tag{13}$$

The Kalman gain is

$$\mathrm{K}_{i,k}^j = \hat{\mathrm{P}}_{i,k}^j(\mathrm{H}_{i,k}^j)^T \mathrm{R}(s_{i,k}^j)^{-1} \tag{14}$$

and $\mathrm{H}_{i,k}^j = \frac{\partial h_i}{\partial \mathbf{x}}|_{\mathbf{x}=\hat{\mathbf{x}}_{k|k-1}^j}$. In LOS conditions, $m(s_{i,k}^j) = 0$ and $\mathrm{R}(s_{i,k}^j) = \sigma_n^2$, while in NLOS, $m(s_{i,k}^j) = \mu_{\mathrm{NLOS}}^j$ and $\mathrm{R}(s_{i,k}^j) = \eta^j$.

Conditioned upon $\mathbf{s}_{k-1}^j, \mathbf{x}_{k-1}^j$ and $\mathbf{z}_k$, to sample $\{\mathbf{s}_k^j, \theta^j\}$ from $p(\mathbf{s}_k, \theta|\mathbf{z}_{1:k})$, we choose the following trial distribution:

$$\begin{aligned} &q(\mathbf{s}_k, \theta|\mathbf{s}_{k-1}^j, \mathbf{x}_{k-1}^j, \mathbf{z}_{1:k}) \\ &= \mathrm{P}(\mathbf{s}_k|\mathbf{s}_{k-1}^j, \mathbf{x}_{k-1}^j, \theta, \mathbf{z}_k)p(\theta|\mathbf{s}_{k-1}^j, \mathbf{x}_{k-1}^j, \mathbf{z}_{k-1}) \\ &= \frac{p(\mathbf{z}_k|\mathbf{s}_k, \mathbf{x}_{k-1}^j, \theta)\mathrm{P}(\mathbf{s}_k|\mathbf{s}_{k-1}^j)}{p(\mathbf{z}_k|\mathbf{s}_{k-1}^j, \mathbf{x}_{k-1}^j, \theta)} \times p(\theta|\mathbf{s}_{k-1}^j, \mathbf{x}_{k-1}^j, \mathbf{z}_{k-1}) \end{aligned} \tag{15}$$

where $p(\mathbf{z}_k|\mathbf{s}_k, \mathbf{x}_{k-1}^j, \theta)$ can be further approximated as:

$$\begin{aligned} p(\mathbf{z}_k|\mathbf{s}_k, \mathbf{x}_{k-1}^j, \theta) &= \int p(\mathbf{z}_k|\mathbf{s}_k, \mathbf{x}_k, \theta)p(\mathbf{x}_k|\mathbf{x}_{k-1}^j)d\mathbf{x}_k \\ &\approx \int p(\mathbf{z}_k|\mathbf{s}_k, \mathbf{x}_k, \theta)\delta(\mathbf{x}_k - \hat{\mathbf{x}}_{k|k-1}^j)d\mathbf{x}_k \\ &= p(\mathbf{z}_k|\mathbf{s}_k, \hat{\mathbf{x}}_{k|k-1}^j, \theta) \end{aligned} \tag{16}$$

Based on the independent transition of the M sight conditions and using the same point approximation method in (16), the trial distribution for $\mathbf{s}_k$ in (15) can be further expressed as

$$\begin{aligned} &\mathrm{P}(\mathbf{s}_k|\mathbf{s}_{k-1}^j, \mathbf{x}_{k-1}^j, \theta, \mathbf{z}_k) \\ &= \frac{\prod_{i=1}^{M} p(z_{i,k}|\hat{\mathbf{x}}_{k|k-1}^j, s_{i,k}, \theta)\mathrm{P}(s_{i,k}|s_{i,k-1}^j)}{p(\mathbf{z}_k|\mathbf{s}_{k-1}^j, \mathbf{x}_{k-1}^j)}, \end{aligned} \tag{17}$$

The likelihood $p(z_{i,k}|\hat{\mathbf{x}}_{k|k-1}^j, s_{i,k}^j, \theta)$ conforms approximately to a Gaussian distribution with mean $\hat{z}_{i,k|k-1}^j$ (13) and covariance:

$$\hat{\Sigma}_{i,k|k-1}^j = \mathrm{H}_{i,k}^j\hat{\mathrm{P}}_{k|k-1}^j(\mathrm{H}_{i,k}^j)^T + \mathrm{R}(s_{i,k}^j). \tag{18}$$

The corresponding importance weight can be calculated as

$$
\begin{aligned}
w_k^j &\propto w_{k-1}^j p(\mathbf{z}_k|\mathbf{s}_{k-1}^j, \mathbf{x}_{k-1}^j, \theta) \\
&= w_{k-1}^j \sum_{\mathbf{s}_k} \left[p(\mathbf{z}_k|\mathbf{s}_k, \mathbf{x}_{k-1}^j, \theta) \mathrm{P}(\mathbf{s}_k|\mathbf{s}_{k-1}^j) \right] \\
&\approx w_{k-1}^j \sum_{\mathbf{s}_k} \left[\prod_{i=1}^{M} p(z_{i,k}|\hat{\mathbf{x}}_{k|k-1}^j, s_{i,k}, \theta) \mathrm{P}(s_{i,k}|s_{i,k-1}^j) \right].
\end{aligned} \tag{19}
$$

To infer the parameter θ, we first specify on them the Gaussian inverse chi-square prior, which is conjugate prior distribution and has computational convenience [13].

Suppose at time t_{k-1}, $p(\theta|\mathbf{s}_{k-1}^j, \mathbf{x}_{k-1}^j, \mathbf{z}_{1:k-1}) = \mathrm{N-Inv}-\chi^2(\breve{\mu}_{k-1}^j, \breve{\kappa}_{k-1}^j, \breve{\nu}_{k-1}^j, \breve{\eta}_{k-1}^j)$, that is,

$$
\begin{aligned}
p(\mu_{\mathrm{NLOS}}|\eta, \mathbf{s}_{k-1}^j, \mathbf{x}_{k-1}^j, \mathbf{z}_{1:k-1}) &= \mathrm{N}(\breve{u}_{k-1}^j, \eta/\breve{\kappa}_{k-1}^j) \\
p(\eta|\mathbf{s}_{k-1}^j, \mathbf{x}_{k-1}^j, \mathbf{z}_{1:k-1}) &= \chi^{-2}(\breve{\nu}_{k-1}^j, \breve{\eta}_{k-1}^j)
\end{aligned} \tag{20}
$$

At the end of time t_k, the trial sampling density for θ is updated according to

$$
\begin{aligned}
p(\theta|\mathbf{x}_k^j, \mathbf{s}_k^j, \mathbf{z}_{1:k}) &\propto p(\mathbf{z}_k|\theta, \mathbf{x}_k^j, \mathbf{s}_k^j) p(\theta|\mathbf{x}_{k-1}^j, \mathbf{s}_{k-1}^j, \mathbf{z}_{1:k-1}) \\
&= \mathrm{N-Inv}-\chi^2(\breve{\mu}_k^j, \breve{\kappa}_k^j, \breve{\nu}_k^j, \breve{\eta}_k^j)
\end{aligned} \tag{21}
$$

where the hyperparameters $\{\breve{\mu}_k^j, \breve{\kappa}_k^j, \breve{\nu}_k^j, \breve{\eta}_k^j\}$ can be explicitly derived in terms of the prior parameters and the sufficient statistics of the data:

$$
\begin{aligned}
\breve{\mu}_k^j &= \frac{\breve{\kappa}_{k-1}^j}{\breve{\kappa}_{k-1}^j + n_k^j} \breve{\mu}_{k-1}^j + \frac{1}{\breve{\kappa}_{k-1}^j + n_k^j} \sum_{i=1}^{n_k^j} \epsilon_{i,k}^j \\
\breve{\kappa}_k^j &= \breve{\kappa}_{k-1}^j + n_k^j \\
\breve{\nu}_k^j &= \breve{\nu}_{k-1}^j + n_k^j \\
\breve{\nu}_k^j \breve{\lambda}_k^j &= \breve{\nu}_{k-1}^j \breve{\lambda}_{k-1}^j + \sum_{i=1}^{n_k^j} (\epsilon_{i,k}^j - \bar{\epsilon}_k^j)^2 \\
&\quad + \frac{\breve{\kappa}_{k-1}^j n_k^j}{\breve{\kappa}_{k-1}^j + n_k^j} (\bar{\epsilon}_k^j - \breve{\mu}_{k-1}^j)^2 \\
\epsilon_{i,k}^j &= (z_{i,k} - h_i(\mathbf{x}_k^j)) \cdot \delta(s_{i,k}^j - 1) \\
\bar{\epsilon}_k^j &= \frac{1}{n_k^j} \sum_{i=1}^{n_k^j} \epsilon_{i,k}^j
\end{aligned} \tag{22}
$$

and n_k^j is the number of the NLOS in $\mathbf{s}_k^j$.

In brief, the particle filtering is first applied to get the density estimation of sight condition, in which $\mathbf{s}_k^j$ is sampled by the optimal trial distribution to achieve the minimum conditional variance of importance weight. Then, the decentralized EKF method is used to get the mean $\hat{\mathbf{x}}_k^j$ and covariance $\hat{\mathrm{P}}_k^j$. In the parameter learning step, the posterior for the unknown parameter $p(\theta|\mathbf{x}_k^j, \mathbf{s}_k^j, \mathbf{z}_{1:k})$ is further updated based on $p(\theta|\mathbf{x}_{k-1}^j, \mathbf{s}_{k-1}^j, \mathbf{z}_{1:k-1}), \mathbf{x}_k^j, \mathbf{s}_k^j$, and $\mathbf{z}_k$. The proposed method of RBPF with parameter learning (RBPF-PL) is summarized in Algorithm 1.

Algorithm 1: RBPF-PL

for $k = 1, 2, \ldots$ **do**
  for $j = 1, 2, \ldots, N$ **do**
    Compute predicted mean $\hat{\mathbf{x}}_{k|k-1}^j$ and covariance $\hat{\mathrm{P}}_{k|k-1}^j$ using (11),(12) and new weight w_k^j using (19).
  end for
  Resample particles $\{w_k^j, \mathbf{s}_{k-1}^j, \hat{\mathbf{x}}_{k|k-1}^j, \hat{\mathrm{P}}_{k|k-1}^j, \breve{\mu}_{k-1}^j, \breve{\kappa}_{k-1}^j, \breve{\nu}_{k-1}^j, \breve{\eta}_{k-1}^j, \theta^j\}_{j=1}^N$ using new weights w_k^j to obtain $\{w_k^l, \mathbf{s}_{k-1}^l, \hat{\mathbf{x}}_{k|k-1}^l, \hat{\mathrm{P}}_{k|k-1}^l, \breve{\mu}_{k-1}^l, \breve{\kappa}_{k-1}^l, \breve{\nu}_{k-1}^l, \breve{\eta}_{k-1}^l, \theta^l\}_{l=1}^N$, where $w_k^{(l)} = \frac{1}{N}$.
  for $l = 1, 2, \ldots, N$ **do**
  1. Sample $\mathbf{s}_k^l$ according to (17).
  2. Update $\{\hat{\mathbf{x}}_k^l, \hat{\mathrm{P}}_k^l\}_{l=1}^N$ according to (9)-(14).
  3. Update hyperparameters $\{\breve{\mu}_k^l, \breve{\kappa}_k^l, \breve{\nu}_k^l, \breve{\eta}_k^l\}$ according to (22).
  4. Sample θ^l according to (21).
  end for
end for

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5 Simulation results

The mobile trajectories are generated according to the motion model described in Section 3. Signals from three BSs are assumed to be received at every epoch. The random acceleration σ_x, σ_y are both chosen to 0.5 m/s^2. The simulated trajectory has $L = 1600$ time samples, and the sample interval $\Delta t = 0.5$ s. The measurement data are generated by adding the measurement noise and the NLOS noise to the true distance from MS to each BS. The measurement noise is assumed to be a white random variable with zero mean and standard deviation $\sigma_n = 150$ m, whereas the NLOS measurement noise is also assumed to be a white random variable but with positive mean $\mu_{\mathrm{NLOS}} = 513$ m and standard deviation $\sigma_{\mathrm{NLOS}} = 409$ m [14]. The mode transition probability is chosen by $p_0 = p_1 = 0.8$. The LOS or NLOS mode between the MS with each BS is generated by sampling from the transition probability of the Markov chain, and is changed every 10 samples in each transition case. The initial estimation of the mode probability are set to $p(s_{i,0} = 0) = p(s_{i,0} = 1) = 0.5$, where $i = 1,\ 2$ and 3. The initial value for the hyperparameters are set as $\{\breve{\mu}_0 = 1000, \breve{\kappa}_0 = 1, \breve{\nu}_0 = 1, \breve{\eta}_0 = (5\sigma_n)^2\}$, which represents very 'vague' prior information on NLOS parameter θ. All the simulation results are obtained based on $n_{\mathrm{MC}} = 20$ Monte Carlo realizations with the same parameters.

We compare the performances of the proposed RBPF-PL and the standard particle filtering with parameter learning (PF-PL) mentioned in Section 3. To evaluate the performances that could be achieved, we also give the estimation results under the following two assumptions: (1) by assuming the sight conditions known for the whole trajectory, RBPF-PL is used only to infer $\{\mathbf{x}_k, \theta\}$ (RBPF-PL($\mathbf{s}_k$ known)). (2) by assuming NLOS parameters θ known, RBPF method is used to infer $\{\mathbf{x}_k, \mathbf{s}_k\}$, which would not consider the parameter learning step. For PF-PL, 1000 particles are used, while for other algorithms only 10 particles are used.

Define root square error (RSE) as RSE $\triangleq \sqrt{(\hat{x}_k - x_k)^2 + (\hat{y}_k - y_k)^2}$, the position root mean square error (RMSE) at time t_k as: RMSE$_k$ $\triangleq \sqrt{\frac{1}{n_{\mathrm{mc}}} \sum_{m=1}^{n_{\mathrm{mc}}} [(\hat{x}_{k,m} - x_k)^2 + (\hat{y}_{k,m} - y_k)^2]}$. We present the cumulative distribution function (CDF) of RSE in Fig. 1, the RMSE in Fig. 2 and one realization of parameter learning in Fig. 3.

From Fig. 1-3, RBPF-PL with $\mathbf{s}_k$ known has the best performance. The reason is that, in the proposed algorithm, the sufficient statistics for updating θ and the mobile state inference for $\mathbf{x}_k$ are largely dependent on the density estimation of $\mathbf{s}_k$. When the sight conditions are known during the whole trajectory, the algorithm could have more accurate estimation on NLOS parameter θ, which further improves the estimation for the mobile state .

RBPF has better performance than RBPF-PL, which is reasonable, since in RBPF, the NLOS parameter is known and the parameter learning step is not included. But, the improvement is slight, as shown in Fig. 1 and 2. Combined with the results of Fig. 3, it is clear that the proposed RBPF-PL could effectively estimate the unknown mean and variance to their true value.

Although using 100 times more particles and having the most computation complexity, the PF-PL has the worst accuracy among all the algorithms, which suggests that the prior transition as the trial sampling distribution is not effective to get the fittest particles in mobile tracking and parameter learning.

From the simulation results, it can be concluded that the accurate estimation of sight condition has an important effect on the ultimate precision of mobile tracking and parameter learning. By applying the optimal trial distribution to sample the posterior distribution of the sight conditions, the RBPF-PL method is effective to infer the unknown NLOS parameter and achieves a good tracking performance with small number of particles.

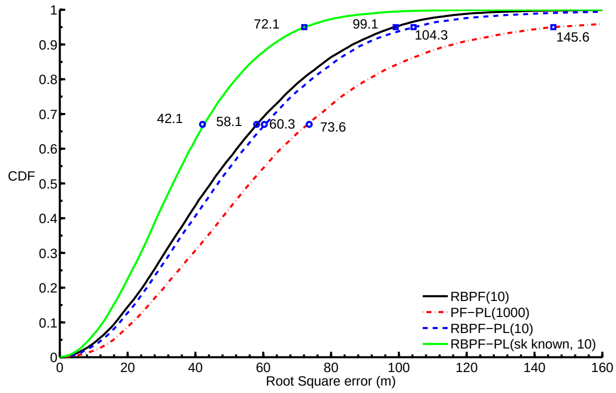

Figure 1: CDF of RSE

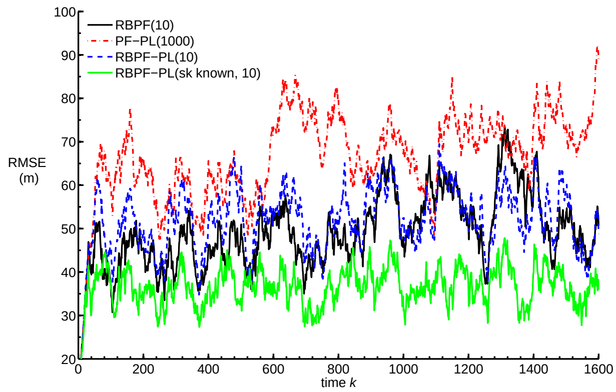

Figure 2: Position RMSE vs. Time instant

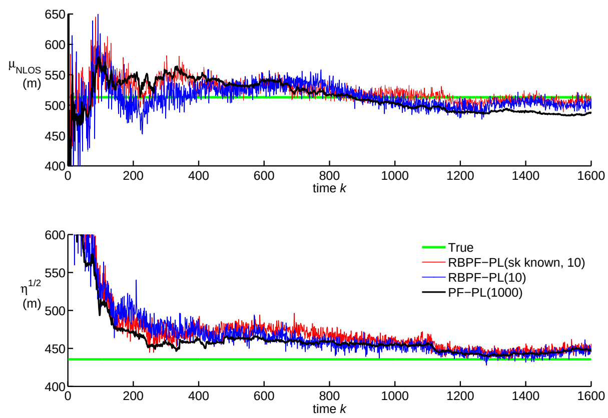

Figure 3: One realization of parameter learning

6 Conclusions

A RBPF-PL method is proposed to track mobility in the mixed LOS/NLOS conditions, where the statistical parameter of NLOS error is unknown. The method first estimates the sight condition using particle fitlers. In order to achieve the minimum weight conditional variance of importance weight and get more accurate estimation of sight condition, the optimal trial distribution is used. Then, by applying decentralized EKF method, the mobile state could be analytically computed. In the parameter learning step, the posterior of the unknown parameter is further updated according to the measurement and the estimation on the sight conditions and mobile state. Simulation results show that, using 10 particles, the RBPF-PL method could achieve a good tracking performance and the NLOS parameters can be effectively inferred.

7 Acknowledgment

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